

# Comment on the $\Theta^+$ width and mass

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(Dated: May 18, 2004)

We discuss the relatively low mass and narrow width prediction for the exotic baryon  $\Theta^+$ , and comment on recent statements by R.L. Jaffe on the subject. We reaffirm that a narrow width of 3.6 – 11.2 MeV follows from the equations of our 1997 paper.

PACS numbers: 12.38.-t, 12.39.-x, 12.39.Dc, 14.20-c

In the 1997 paper [1] we predicted a relatively light ( $M \approx 1530$  MeV) and narrow ( $\Gamma \leq 15$  MeV) exotic baryon with strangeness +1, isospin zero and spin-parity  $\frac{1}{2}^+$ . The paper was published at a time when all previous searches of exotic baryons for thirty years were in vain [2], and the latest phase shift analysis [3] summarizing the  $KN$  scattering data showed no signs of a resonance in this energy range. The prediction motivated and oriented new experimental searches, and in the end of 2002 the first independent observations of the exotic baryon in  $\gamma C$  [4] and  $K^+Xe$  [5] reactions were announced, followed by important confirmation in about ten experiments by spring 2004 [6, 7, 8, 9, 10, 11, 12, 13]. In one year,  $\Theta^+$ , as it has been named following our suggestion [14], made it to the Review of Particle Properties [15]. At the same time, there have been several experiments where  $\Theta^+$  has not been seen [16, 17]. Therefore, one is now looking forward to the next tour of dedicated experiments with higher statistics, for the issue to be finally resolved.

Not unnaturally, theoretical [18, 19, 20, 21, 22], phenomenological [23, 24] and arithmetical [25] aspects of the 1997 paper have been recently scrutinized. In the latter comment, Jaffe states that we have made an arithmetic mistake in Ref. [1] and that, being corrected, it would lead to a prediction of 30 MeV for the  $\Theta$ 's width. Although the questions of the  $\Theta$ 's mass and width have been already discussed in great detail by Ellis, Karliner and Praszalowicz [23] who have basically confirmed the calculations of Ref. [1] and explored the unbiased theoretical and phenomenological uncertainties, we feel it necessary to respond directly to Jaffe's comment. To that end, we have first of all to remind briefly the calculation of the  $\Theta$  width.

The equation (56) for  $\Theta$ 's width from Ref. [1] reads:

$$\Gamma_\Theta = \frac{3 G_{10}^2}{2\pi[m_N + m_\Theta]^2} \frac{m_N}{m_\Theta} \frac{1}{5} |\mathbf{p}|^3 \quad (1)$$

where  $\mathbf{p}$  is the kaon momentum in the decay  $\Theta^+ \rightarrow NK$ , and we have neglected the small correction due to the antidecuplet-octet mixing. The pseudoscalar antidecuplet-octet transition constant  $G_{10}$  is expressed through the more fundamental symmetry constants

$G_{0,1,2}$  (Table 2 of Ref.[1]):

$$G_{10} = G_0 - G_1 - \frac{1}{2}G_2. \quad (2)$$

The constant  $G_2$  is small as it is related to the singlet nucleon axial constant  $g_A^{(0)}$ , better known as the fraction of nucleon spin carried by quarks' spin (equation (53) of [1]):

$$G_2 = \frac{2m_N}{3F_\pi} g_A^{(0)} \simeq 2 \quad (3)$$

if  $g_A^{(0)} \simeq 0.3 \pm 0.1$  is used [26]. Another combination of  $G_{0,1,2}$  determines the standard pion-nucleon coupling constant (equation (50) of [1]):

$$g_{\pi NN} = \frac{7}{10} \left( G_0 + \frac{1}{2}G_1 + \frac{1}{14}G_2 \right) \simeq 13.3 \quad (4)$$

where we have substituted the present value of the pseudoscalar pion-nucleon constant [27]. Eqs.(3,4) allow one to find the combination

$$G_{10} = G_0 + \frac{1}{2}G_1 \simeq 18.9 \quad (5)$$

from phenomenology, but not  $G_{10}$  determining the  $\Theta$  decay. Therefore, in Ref. [1] we have used an additional theoretical input,

$$\frac{G_1}{G_0} = \rho \quad (6)$$

with  $\rho$  ranging from 0.4 to 0.6 as it follows from the estimates in the chiral quark soliton model [28]. Eqs.(5,6) fix the  $G_{10}$  constant needed to compute the  $\Theta$  width. In Ref. [1] we have used the following values of the masses:  $m_\Theta = 1530$  MeV,  $m_N = 940$  MeV,  $m_K = 495$  MeV leading to the kaon momentum in the decay

$$\begin{aligned} |\mathbf{p}| &= \frac{\sqrt{m_\Theta^4 + m_N^4 + m_K^4 - 2m_\Theta^2 m_N^2 - 2m_\Theta^2 m_K^2 - 2m_N^2 m_K^2}}{2m_\Theta} \\ &= 254 \text{ MeV}. \end{aligned} \quad (7)$$

Putting these numbers into eq. (1) one gets finally

$$\Gamma_\Theta = \begin{cases} 3.6 \text{ MeV}, & \rho = 0.6, \\ 6.7 \text{ MeV}, & \rho = 0.5, \\ 11.2 \text{ MeV}, & \rho = 0.4. \end{cases} \quad (8)$$

In view of the theoretical uncertainty in the estimate of the  $G_1/G_0$  ratio, we have concluded that the exotic baryon width must be “less than 15 MeV”, and put it in the Abstract in Ref. [1]. In the original paper we have neglected the small  $G_2$  in the final estimate (8) [52] but took into account the small correction from antidecuplet-octet mixing and hence obtained a slightly higher upper limit,  $\Gamma_\Theta \leq 15$  MeV at  $\rho = 0.4$ . We noted however (before eq.(56)) that it was a conservative estimate and that  $\Theta$  could be more narrow. The key element in the narrow width prediction is the strong cancellation between the  $G_0, G_1$  and  $G_2$  contributions, which we have noticed. Furthermore, we have noticed that in the non-relativistic limit implying  $G_1/G_0 = 4/5$ ,  $G_2/G_0 = 2/5$  one gets zero  $G_{10}$  and hence zero  $\Theta$  width.

A separate issue is the widths of the usual  $(\mathbf{10}, \frac{3}{2})$  baryons. They are not directly related to the above estimate of the  $\Theta$  width, however their discussion has been included in Ref. [1] for completeness. For spin 1/2 decays, there is only one formfactor involved, and in whatever way one treats kinematical factors in the case of  $(\mathbf{8}, \frac{1}{2})$  baryons, one can repeat the same for the  $(\overline{\mathbf{10}}, \frac{1}{2})$  ones. For spin 3/2 decays, it becomes more ambiguous. In the academic limit of large number of colors  $N_c$ , the decuplet-to-octet transition constant  $G_{10}$  is determined by eq. (5) and thus related to the octet  $g_{\pi NN}$  pseudoscalar coupling. However, in reality an additional uncertainty arises for spin  $\frac{3}{2} \rightarrow \frac{1}{2}$  decay widths: Should one use Adler’s formfactors [29], treat the spin 3/2 in the relativistic Rarita–Schwinger formalism and take the exact spin 3/2 density matrix to compute the phase volume, or should one rather estimate the transition matrix element by non-relativistic formulae and then simply multiply it by the relativistic phase volume? Should the symmetry relation (5) be imposed on the axial vector constants or rather on the pseudoscalar ones? Depending on the choice one makes, one gets different functions of the mass ratio  $m_1/m_2$  in the expressions for the spin 3/2 widths ( $m_1$  is the initial and  $m_2$  is the final baryon mass in the decay). This mass ratio is unity in the large  $N_c$  limit, since in this limit both  $N$  and  $\Delta$  are infinitely heavy non-relativistic particles, such that it does not matter which way one decides to resolve the ambiguities, but in the real world it does matter since  $\Delta$  is 30% heavier than the nucleon. This ambiguity is encountered by all people who have attempted to fit the decuplet decays from the knowledge of the  $g_{\pi NN}$  constant, be it from large- $N_c$  or non-relativistic quark considerations [30]. There are infinitely many ways how one can resolve this ambiguity, and any of them is guess work from the strict theory point of view as it corresponds to some particular hypothesis how to sum up an infinite series of unknown corrections in quark masses and  $1/N_c$ . It reflects the true situation and the actual theoretical accuracy with which one is able to compute the spin 3/2 widths from the large- $N_c$  considerations.

When working on the 1997 paper, we have noticed that if, for the spin 3/2 decays, one rescales the symmetry

relation (5), by the  $m_1/m_2$  ratio, the *four* known decuplet decay rates are described very satisfactory and are in accordance with the value of the  $g_{\pi NN}$  constant. Indeed, with this rescaling eqs.(42-45) of Ref. [1] should read:

$$\begin{aligned}\Gamma(\Delta \rightarrow N\pi) &= \frac{3G_{10}^2}{2\pi(m_\Delta + m_N)^2} |\mathbf{p}|^3 \frac{m_\Delta}{m_N} \cdot \frac{1}{5} \\ &= 110 \text{ MeV vs } 100 - 125 \text{ MeV (exp.)}, \\ \Gamma(\Sigma^* \rightarrow \Lambda\pi) &= \frac{3G_{10}^2}{2\pi(m_{\Sigma^*} + m_\Lambda)^2} |\mathbf{p}|^3 \frac{m_{\Sigma^*}}{m_\Lambda} \cdot \frac{1}{10} \\ &= 30.5 \text{ MeV vs } 32.6 \text{ MeV (exp.)}, \\ \Gamma(\Sigma^* \rightarrow \Sigma\pi) &= \frac{3G_{10}^2}{2\pi(m_{\Sigma^*} + m_\Sigma)^2} |\mathbf{p}|^3 \frac{m_{\Sigma^*}}{m_\Sigma} \cdot \frac{1}{15} \\ &= 3.7 \text{ MeV vs } 4.4 \text{ MeV (exp.)}, \\ \Gamma(\Xi^* \rightarrow \Xi\pi) &= \frac{3G_{10}^2}{2\pi(m_{\Xi^*} + m_\Xi)^2} |\mathbf{p}|^3 \frac{m_{\Xi^*}}{m_\Xi} \cdot \frac{1}{10} \\ &= 8.8 \text{ MeV vs } 9.3 \text{ MeV (exp.)},\end{aligned}\quad (9)$$

where  $\mathbf{p}$  is the pion momentum and  $G_{10} \simeq 18.9$  from eq. (5) is used [53]. It is interesting that if one considers only non-strange baryons, the large- $N_c$  relation between the  $\Delta$  width and the  $g_{\pi NN}$  constant for two flavors is well satisfied without the rescaling of the spin-3/2 constant by the mass ratio [31]. It shows once more that there is some arbitrariness in the theoretical treatment of the strange quark mass and  $1/N_c$  corrections to the decuplet decays. Unfortunately, in the write-up a year later after the actual calculations, we did not discuss the problem of the spin-3/2 decays (which was anyhow secondary to the more important issues related to the suggested new spin-1/2 antidecuplet of baryons) and wrote all equations universally as if they were for spin-1/2 decays, but left the numerical values of the widths in the right hand sides computed from the rescaled formulae. Weigel discovered this inconsistency [32] and communicated it to one of us (M.P.) who acknowledged the mistake. We apologize to those who might have been lead into confusion. However, we told very many people about this mistake, including the authors of Ref. [23] and Jaffe.

In his comment [25], Jaffe suggests that one has to take our mistake in the analytical expressions for the decuplet decays at face value, fit the  $\Delta$  decay with a larger value of  $G_{10} \simeq 25$ , get an *unacceptably large*  $g_{\pi NN} \simeq 17.5$  from eq. (4) and correspondingly a larger value of the  $\Theta$  width. However, this is not the way to proceed. If the  $g_{\pi NN}$  constant matches the spin-3/2 decuplet decays, one can use *either*  $g_{\pi NN}$  *or* the decuplet widths as an input to estimate the width of  $\Theta^+$ , since it gives the same. This is the case when one rescales  $G_{10}$  by the  $m_1/m_2$  mass ratio, as in eqs. (9), or does not rescale the constant but uses the 2-flavor relation between  $g_{\pi NN}$  and  $g_{\pi N\Delta}$  [31]. If one does not succeed to match them, one uses the phenomenological value of the spin-1/2  $g_{\pi NN}$  constant to get the same small width of the spin-1/2  $\Theta^+$  as described above, but faces an unrelated problem how to explain the theoretically more dubious spin-3/2 decays.

Let us emphasize it again: The narrow width prediction for the  $\Theta^+$  can be obtained without even mentioning the decuplet. It is founded on the dynamical cancellation in the  $G_{\overline{10}}$  constant, see above [54]. In fact the present day theoretical uncertainty in how “deep” is this cancellation, resulting in the spread in eq. (8), is greater than the theoretical uncertainty in the decuplet decays [55].

We would like to comment on two other statements by Jaffe [25]. Both comments are historic but elucidate physics as well. 20 years ago, in the Fall of 1983 a seminal paper by Witten appeared [34], where there was a brief Note Added in Proof with the now famous quantization condition that only those  $SU(3)$  baryon multiplets appear as rotational states of a chiral soliton in the 3-flavor space, which have hypercharge  $Y = N_c/3 = 1$ , and that the spin of the allowed multiplet is related to the number of particles with that hypercharge. Since no derivation was given, several groups [35] derived this result in 1984–85 in their own manner [56], including two of the present authors [36]. In February 1984 one of us (D.D.) gave lectures on this particular subject at the ITEP Winter School, where the quantization of ordinary and  $SU(3)$  flavor rotations of a chiral soliton has been exhaustively explained and its implications for higher rotational states discussed. In the published version of those lectures D.D. and V.P. have written on p. 90: “We thus come to the conclusion that the lowest states of the chiral soliton are the octet with spin 1/2 and the decuplet with spin 3/2. We leave it as an exercise to the reader to find other multiplets which can be interpreted as rotational states of a spherically symmetric chiral soliton.” When one knows Witten’s quantization conditions, one opens any book on  $SU(3)$  and observes that the next baryon multiplets satisfying them are  $(\overline{10}, \frac{1}{2})$ ,  $(27, \frac{3}{2})$ ,  $(27, \frac{1}{2})$  and so on. In 1984 the potential existence of an exotic *antidecuplet* of baryons became plain to the skyrmion community.

Knowing about the potential existence of the baryon antidecuplet, many people including ourselves made estimates of its masses as early as 1984 with whatever calculational tools available at that time. In his comment, Jaffe gives credit to Praszalowicz for predicting the correct mass of the  $\Theta$ . Indeed, Praszalowicz mentioned the mass of the lowest baryon, the future  $\Theta$ , at 1530 MeV [37] as following from the Skyrme model. However, it may be worth pointing out that there were inconsistencies in that calculation. First of all, in the unrealistic Skyrme model one has to make a difficult choice between having the nucleon and  $\Delta$  masses correct and the  $F_\pi$  constant completely wrong ( $F_\pi \simeq 64$  MeV vs 93 MeV (exp.)) [31, 37], or *vice versa*. To make matters worse, in the Skyrme model the normal term proportional to strangeness in the baryon splitting, is absent. Therefore, to account approximately for the observable splitting in strangeness in the octet and decuplet baryons, one has to employ the Yabu–Ando method of involving higher corrections in the strange quark mass, which adds new free parameters. As seen from Fig. 1 in Ref. [37] the choice of a free parameter corresponding to  $m_\Theta = 1530$  MeV, cor-

responds also to poor masses of the normal  $N, \Lambda$  and  $\Sigma$ . If one makes a better fit to the known baryons,  $\Theta$  shifts to 1340 MeV, i.e. below the threshold for strong decays. The  $F_\pi$  constant remains 1.5 times less than it should be. If anything, in this very useful paper Praszalowicz demonstrated that the Skyrme model was unfit to make accurate predictions. As the author correctly noted himself, “one has to express criticism against [the Yabu–Ando method], as it sums up an arbitrary subseries of the strange quark masses, neglecting other terms of the same order” [38]. The same remark concerns the estimate of  $m_\Theta$  by Walliser who, using basically the same Skyrme model but another version of the  $SU(3)$  symmetry breaking, obtained it at about 1700 MeV [39]. In another variant of the Skyrme model Walliser got a remarkable  $m_\Theta = 1550$  MeV! The earliest printed estimate we found was in the 1984 paper by Biedenharn and Dothan [40] who evaluated the antidecuplet mass (without splitting) at  $m_N + 600$  MeV = 1540 MeV, with the conclusion that it was an artifact of the model. In short, one could get various “predictions” for the  $\Theta$  mass from the Skyrme model, depending on what observables for the established hadrons one was prepared to sacrifice. Since there were too many inherent inconsistencies inside the model, none of these authors seemed to have taken the antidecuplet for real.

As to the widths of the antidecuplet baryons, in the standard version of the Skyrme model the constants  $G_{1,2}$  discussed above are zero. There is no possibility for the dynamical cancellation of the  $G_0$  constant, leading to the narrow  $\Theta$  width, which therefore could have never been obtained in the Skyrme model.

It was not until the modern chiral quark soliton model of baryons [41] has been developed that one could estimate the masses and the widths of the antidecuplet baryons in a consistent way. The instanton-based chiral quark model gives a coherent picture of mesons and baryons; in particular it explains the basic facts about baryons, which are mysterious in the conventional constituent quark models: why quark spins carry only 1/3 of the nucleon spin [42], why the nucleon sigma term is 4 times larger than counted from quarks [43], why there are many antiquarks in the nucleon at a low virtuality [44], why the sea antiquarks are flavor-asymmetric [45], and many other features, not to mention an overall fair description of masses, magnetic moments and formfactors [46]. Simultaneously, it gives the reason why constituent quark models are in many cases successful. There are basically no free parameters in the model as it follows from the QCD lagrangian [47]. When one feels that the known basic facts are understood, one may risk to make a prediction, despite a heavy pressure from the unsuccessful attempts to find exotic baryons in the past.

The accurate prediction of the  $\Theta$  mass in Ref. [1] was to some extent a luck. It was in part based on the use of a certain value of the nucleon sigma term resulting in a large splitting in the antidecuplet, and on identifying the

$N(1710)$  resonance as the antidecuplet member. These were legitimate assumptions in 1997 but later experimental data shifted the sigma term to larger values [24, 48], resulting in a smaller antidecuplet splitting [49], while a new analysis [50] indicated that the former  $N(1710)$  might be in fact lighter [57]. It seems like there is a lucky cancellation between the inaccuracies in the two inputs, each on the scale of a few tens MeV.

The important points of Ref. [1] were a) that the exotic baryon  $\Theta^+$  with spin-parity  $\frac{1}{2}^+$  must exist, b) that

it must be relatively light, c) that it must be narrow. These points came from the experience in the quantitative description of the properties of the usual hadrons. The relatively low mass and width of the antidecuplet are explained in qualitative terms in Ref. [49].

We thank K. Goeke, R. Jaffe, K. Hicks, B. Holstein, M. Karliner, M. Praszalowicz and M. Strikman for useful comments on the first version.

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- [1] D. Diakonov, V. Petrov and M. Polyakov, Z. Phys. A **359**, 305 (1997), [hep-ph/9703373](#).
  - [2] Particle Data Group, mini-review in the 1986 edition.
  - [3] J.S. Hyslop, R.A. Arndt, L.D. Roper and R.L. Workman, Phys. Rev. D **46**, 961 (1992).
  - [4] T. Nakano (LEPS Collaboration), Talk at the PANIC 2002 (Oct. 7, 2002, Osaka); T. Nakano et al., Phys. Rev. Lett. **91**, 012002 (2003), [hep-ex/0301020](#).
  - [5] V.A. Shebanov (DIANA Collaboration), Talk at the Session of the Nuclear Physics Division of the Russian Academy of Sciences (Dec. 3, 2002, Moscow); V.V. Barmin, A.G. Dolgolenko et al., Phys. Atom. Nucl. **66**, 1715 (2003) [Yad. Fiz. **66**, 1763 (2003)], [hep-ex/0304040](#).
  - [6] S. Stepanyan, K. Hicks et al. (CLAS Collaboration), Phys. Rev. Lett. **91**, 252001 (2003); V. Kubarovsky et al. (CLAS Collaboration), Phys. Rev. Lett. **92**, 032001 (2004), [hep-ex/0311046](#).
  - [7] J. Barth et al. (SAPHIR Collaboration), Phys. Lett. B **572**, 127 (2003), [hep-ex/0307083](#).
  - [8] A.E. Asratyan, A.G. Dolgolenko and M.A. Kubantsev, submitted to Yad. Fiz., [hep-ex/0309042](#).
  - [9] A. Airapetian et al. (HERMES Collaboration), Phys. Lett. B **585**, 213 (2004), [hep-ex/0312044](#).
  - [10] A. Aleev et al. (SVD Collaboration), [hep-ex/0401024](#).
  - [11] M. Abdel-Bary et al. (COSY-TOF Collaboration), [hep-ex/0403011](#).
  - [12] P.Zh. Aslanyan, V.N. Emelyanenko, G.G. Rikhhvitzkaya, [hep-ex/0403044](#).
  - [13] S. Chekanov et al. (ZEUS Collaboration), [hep-ex/0403051](#).
  - [14] D. Diakonov, e-letter to parties involved (April 12, 2003).
  - [15] Particle Data Group, Tentative Review of Particle Properties (2004).
  - [16] J.Z. Bai et al. (BES Collaboration), [hep-ex/0402012](#).
  - [17] K.T. Knopfle, M. Zavertyaev and T. Zivko (HERA-B Collaboration), [hep-ex/0403020](#).
  - [18] T. Cohen, Phys. Lett. B **581**, 175 (2004), [hep-ph/0309111](#).
  - [19] D. Diakonov and V. Petrov, Phys. Rev. D **69**, 056002 (2004), [hep-ph/0309203](#).
  - [20] N. Itzhaki, I.R. Klebanov, P. Ouyang and L. Rastelli, Nucl. Phys. B **684**, 264 (2004), [hep-ph/0309305](#).
  - [21] P. Pobylitsa, [hep-ph/0310221](#).
  - [22] M. Praszalowicz, Phys. Lett. B **583**, 96 (2004), [hep-ph/0311230](#); [hep-ph/0402038](#).
  - [23] J. Ellis, M. Karliner and M. Praszalowicz, [hep-ph/0401127](#).
  - [24] P. Schweitzer, [hep-ph/0312376](#).
  - [25] R.L. Jaffe, [hep-ph/0401187v2](#).
  - [26] B.W. Filippone and X. Ji, Adv. Nucl. Phys. **26**, 1 (2001), [hep-ph/0101224](#).
  - [27] M.M. Pavan, R.A. Arndt, I.I. Strakovsky and R.L. Workman, Physica Scripta, **87**, 65 (2000), [nucl-th/9910040](#); T.E.O. Ericson, B. Loiseau and A. Thomas, Phys. Rev. C **66**, 014005 (2002), [hep-ph/0009312](#).
  - [28] A. Blotz, M.V. Polyakov and K. Goeke, Phys. Lett. B **302**, 151 (1993); C. Christov, K. Goeke, V. Petrov, P. Pobylitsa, M. Wakamatsu and T. Watabe, Phys. Lett. B **325**, 467 (1994); A. Blotz, K. Goeke and M. Praszalowicz, Phys. Rev. D **53**, 484 (1996).
  - [29] S. Adler, Ann. Phys. (N.Y.) **50**, 189 (1968).
  - [30] T.R. Hemmert, B.R. Holstein and N.C. Mukhopadhyay, Phys. Rev. D **51**, 158 (1995), [hep-ph/9409323](#).
  - [31] G.S. Adkins, C.R. Nappi, E. Witten, Nucl. Phys. B **228**, 552 (1983).
  - [32] H. Weigel, Eur. Phys. J. A **2**, 391 (1998), [hep-ph/9804260](#).
  - [33] A. Rathke, Diploma thesis, Bochum University (1998).
  - [34] E. Witten, Nucl. Phys. B **160**, 433 (1983).
  - [35] E. Guadagnini, Nucl. Phys. B **236**, 35 (1984); L.C. Biedenharn, Y. Dothan and A. Stern, Phys. Lett. B **146**, 289 (1984); P.O. Mazur, M.A. Nowak and M. Praszalowicz, Phys. Lett. B **147**, 137 (1984); A.V. Manohar, Nucl. Phys. B **248**, 19 (1984); M. Chemtob, Nucl. Phys. B **256**, 600 (1985); S. Jain and S.R. Wadia, Nucl. Phys. B **258**, 713 (1985).
  - [36] D. Diakonov and V. Petrov, *Baryons as solitons*, preprint LNPI-967 (1984), published in *Elementary Particles*, Proc. 12th ITEP Winter School, Energoatomizdat, Moscow (1985) p.50-93 (in Russian).
  - [37] M. Praszalowicz, in *Skyrmions and Anomalies*, M. Jezabek and M. Praszalowicz, eds., World Scientific, Singapore (1987), p. 112, see the commented reprint in Phys. Lett. B **575**, 234 (2003), [hep-ph/0308114](#).
  - [38] M. Praszalowicz, in *Baryons as Skyrme Solitons*, G. Holzwarth, ed., World Scientific, Singapore (1993), p. 43, [hep-ph/9211278](#); citation from the first page.
  - [39] H. Walliser, Nucl. Phys. A **548**, 649 (1992); H. Walliser and V.B. Kopeliovich, J. Exp. Theor. Phys. **97**, 433 (2003) [Zh. Eksp. Teor. Fiz. **124**, 483 (2003)], [hep-ph/0304058](#).
  - [40] L.C. Biedenharn and Y. Dothan, in *From SU(3) to Gravity* (Ne'eman Festschrift), E. Costman and G. Tauber, eds., Cambridge University Press (1986), p. 15 [Duke University preprint (1984)].
  - [41] D. Diakonov and V. Petrov, JETP Lett. **43**, 75 (1986)

- [Pisma ZhETF **43**, 57 (1986)]; D. Diakonov, V. Petrov and P.V. Pobylitsa, Nucl. Phys. B **306**, 809 (1988).
- [42] M. Wakamatsu and H. Yoshiki, Nucl. Phys. A **524**, 561 (1991). The first qualitative explanation of the ‘spin crisis’ from the Skyrme model point of view, namely zero fraction of proton spin carried by quarks’ spin, was given in: S.J. Brodsky, J.R. Ellis and M. Karliner, Phys. Lett. B **206**, 309 (1988).
- [43] D. Diakonov, V. Petrov and M. Praszalowicz, Nucl. Phys. B **323**, 53 (1989).
- [44] D. Diakonov, V. Petrov, P. Pobylitsa, M. Polyakov and C. Weiss, Nucl. Phys. B **480**, 341 (1996), [hep-ph/9606314](#); Phys. Rev. D **56**, 4069 (1997), [hep-ph/9703420](#).
- [45] P.V. Pobylitsa, M.V. Polyakov, K. Goeke, T. Watabe and C. Weiss, Phys. Rev. D **59**, 034024 (1999), [hep-ph/9804436](#).
- [46] C. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, Th. Meissner, E. Ruiz Arriola and K. Goeke, Prog. Part. Nucl. Phys. **37**, 91 (1996), [hep-ph/9604441](#).
- [47] D. Diakonov and V. Petrov, in *Handbook of QCD*, M. Shifman, ed., World Scientific, Singapore (2001), vol. 1, p. 359, [hep-ph/0009006](#); D. Diakonov, Prog. Part. Nucl. Phys. **51**, 173 (2003), [hep-ph/0212026](#).
- [48] M.M. Pavan, R.A. Arndt, I.I. Strakovsky and R.L. Workman, in Proceedings of the 9th International Symposium on Meson-Nucleon Physics and the Structure of the Nucleon (MENU2001), Washington DC, July 2001, H. Haberzettl and W.J. Briscoe, eds., piN Newslett. 16, 110 (2002), [hep-ph/0111066](#).
- [49] D. Diakonov and V. Petrov, to appear in Phys. Rev. D, [hep-ph/0310212](#).
- [50] R.A. Arndt, Ya.I. Azimov, M.V. Polyakov, I.I. Strakovsky and R.L. Workman, Phys. Rev. C **69**, 035208 (2004), [nucl-th/0312126](#).
- [51] V. Kouznetsov, talk at NSTAR-2004 (March 2004).
- [52] In 1997 the fraction of the proton spin carried by quark spins was compatible with zero.
- [53] In the numerics, we have averaged the masses over the isospin components, and took the experimental widths from the PDG-2002 edition, which explains a small deviation in numbers from eqs.(42-45) of the earlier Ref. [1].
- [54] Recently Praszalowicz has demonstrated that this cancellation persists at any number of colors [22].
- [55] It should be mentioned that a more careful analysis of the semileptonic hyperon decays in Ref. [33] lead to the estimate  $\Gamma_{\Theta} < 5 \text{ MeV}$ .
- [56] From this list, Guadagnini, Mazur et al. and Jain and Wadia quantized the  $SU(3)$  skyrmion independently; other authors cite Guadagnini’s paper.
- [57] Preliminary data from the  $\gamma n \rightarrow \eta n$  reaction at GRAAL in Grenoble indicate a narrow  $N^*$  resonance at 1670 MeV whose properties seem to fit its antidecuplet nature [51].